Massive $\longrightarrow$ Massless

$$\lambda_\alpha^{I}\ \hat{\lambda}_{\dot\alpha}^{I} \xrightarrow[\text{H.E}]{} \lambda_\alpha\ \tilde{\lambda}_{\dot\alpha}$$

$SU(2)$ $\quad$ $U(1)\ e^{-\frac{s}{2}}$ $\quad$ $U(1)\ e^{\frac{s}{2}}$

$m \longrightarrow 0$

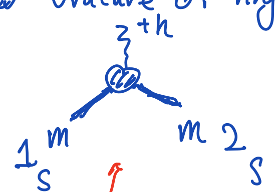
projection $\quad$ 1, 2

$$\lambda_\alpha^{I} = \lambda_\alpha \zeta^{I-} + \cancel{\eta_\alpha \zeta^{I+}}$$

$$\hat{\lambda}_{\dot\alpha}^{I} = \tilde{\lambda}_{\dot\alpha} \zeta^{I+} + \cancel{\hat{\eta}_{\dot\alpha} \zeta^{I-}}$$

$\eta_\alpha \hat{\eta}_{\dot\alpha} \propto m$

① Nature of higher spin massive state.



minimal coupling $\propto x^h(\epsilon^{\alpha_1\beta_1}\epsilon^{\alpha_2\beta_2}\cdots\epsilon^{\alpha_{2s}\beta_{2s}}) + \epsilon\epsilon\cdots\epsilon\frac{\lambda\lambda}{m}x^2$

$\lambda_\alpha^I \lambda_\alpha$

$\rightarrow$ putting back the massive spinor

$$x^h\left(\langle 1^{I_i} 2^{J_i}\rangle\right)^{2S}$$

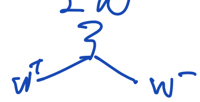
$$x^h\Big(\underbrace{\epsilon\epsilon\cdots\epsilon}_{\text{minimal}} + \epsilon\epsilon\cdots\epsilon\frac{\lambda\lambda x}{m} + \epsilon\cdots\epsilon\left(\frac{\lambda\lambda x}{m}\right)^2 \cdots\Big)$$

(g-2)

Finite size effects

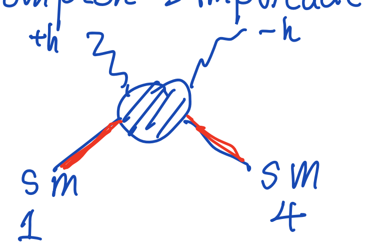
QED $\quad x\langle 12\rangle$

EW

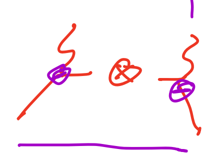


$x\langle 12\rangle^2$

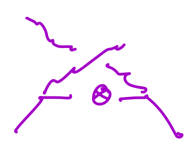
Compton Amplitude



$$A_4(1^s 2^{+h} 3^{-h} 4^s)\Big|_{s=m^2} =$$



$$\Big|_{u=m^2} =$$



$\rightarrow$ for $h=1$ when $S \le 1$ have a local form

$$\frac{(\cdots)}{s\,u} \xleftarrow{HE} \frac{(\cdots)}{(s-m^2)(u-m^2)} + \text{Regular}$$

$s > 1$

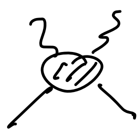
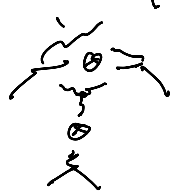
$$\times \quad \frac{(\cdots)}{(s-m^2)(u-m^2)} + \frac{(\quad)}{(s-m^2)(m^2)^a} + \frac{(\quad)}{(u-m^2)(m^2)^c} + \text{Regular}$$

$\sim (m^2)$

obstruction to taking $m^2 \to 0$

setting a fundamental size

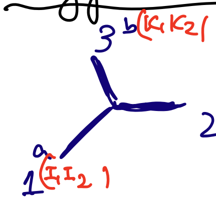
→ for $h=2$ . . . $s \leq 2$

 $=$  $\dfrac{(\cdots)}{(s-m^2)(u-m^2)t}$

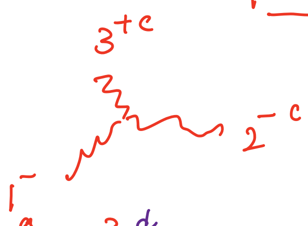
$s > 2$

$$\frac{(\cdots)}{(s-m^2)(u-m^2)t} + \frac{(\cdots)}{(s-m^2)t(m^2)^a} + (s \leftrightarrow u)$$

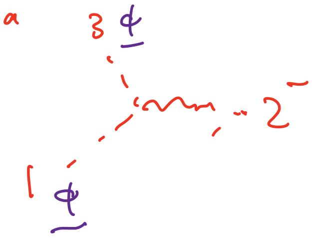
② Higgs-mechanism

spin-1 → 

$$= \boxed{\frac{g f^{abc}}{m_a m_b m_c}\left(\langle 1^{I_1} 2^{J_1}\rangle [1^{I_2} 2^{J_2}] \langle 3^{K_1}| p_1 - p_2 | 3^{K_2}] + \text{cyclic}\right)}$$

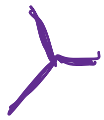
H.E. → 

$$\to g f^{abc} \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$



$$\to \frac{g f^{abc}}{m_a m_c} \frac{\langle 12\rangle\langle 23\rangle}{\langle 31\rangle} (m_b^2 - m_c^2 - m_a^2)$$

Spin-2



$$= \frac{1}{M_{pl}\, m^6}\left[\langle 12\rangle[12]\langle 3|P_1-P_2|3] + \text{cyclic}\right]^2$$

$\rightarrow$ H.E

$(-2\ -2\ +2)$ $\quad \dfrac{1}{M_{pl}} \dfrac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$

$(0\ -2\ 0)$ $\quad \dfrac{2}{M_{pl}} \dfrac{\langle 12\rangle^2\langle 23\rangle^2}{\langle 13\rangle^2}$

equivalence principle

Higgs mech for spin-2 inconsistent with

Bootstrap $(3 \to 4) \to (3 \to n)$

Tree: BCFW recursion

Britto Cachazo Feng Witten

$$q^2 = q\cdot P_1 = q \cdot P_2 = 0$$

$z \to \infty$

$= 0$

$$A_n(\cdots, z) = A_n(\cdots)\Big|_{P_1 \to P_1 + zq,\ P_2 \to P_2 - zq}$$

P_1 P_3

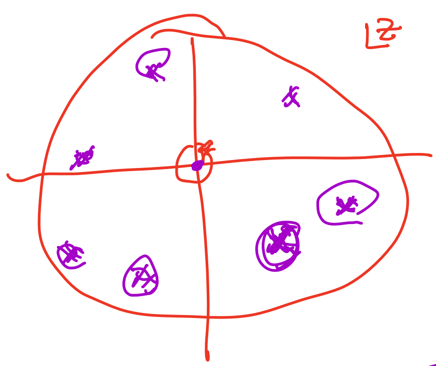
P_3

$P_{<7}$

in 4D

$$q^{\alpha\dot\alpha} = \lambda_1^{\alpha}\tilde\lambda_2^{\dot\alpha} \qquad \lambda_2^{\alpha}\tilde\lambda_1^{\dot\alpha}$$

$$\text{aim} \Rightarrow A_n(\cdots, 0) = \oint \frac{dz}{z} A_n(\cdots, z)$$



$$= -\sum_{\text{residue}} \frac{A_n(\cdots, z)}{z}$$

$A_n(\cdots; z)$

$\nearrow$ rational function

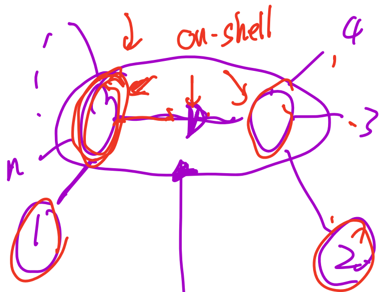
$$= \sum \frac{R}{(P_1+P_2 \cdots P_i)^2}$$

YM $\to$ $(+-)$

GR $\to$

$$(P_1+P_2+P_3+P_4)^2 \leftrightarrow (P_1+\cancel{zq}+P_2-\cancel{zq}+P_3+P_4)^2$$

z



$$\frac{n(z)}{(P_2+P_3+P_4)^2} \to \frac{n(z)}{(P_2+P_3+P_4-zq)^2}$$

$$= \frac{n(z)}{(P_2+P_3+P_4)^2 - z\,2q\cdot(P_2+P_3+P_4)}$$

$$= \frac{n(z)}{2q\cdot K\left(\frac{K^2}{2q\cdot K} - z\right)}$$

$$K = (P_2+P_3+P_4)$$

When z hits the singularity

$$z^* = \frac{K^2}{2(q\cdot K)}$$

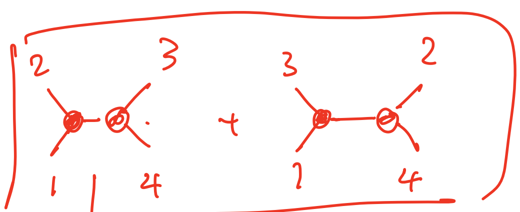
$$n(z)\big|_{z=z^*} = A_{n-2} \times A_4$$

$$A_n = \sum_{dia} \frac{1}{z} \frac{A_L \otimes A_R}{(2q\cdot K)}\Bigg|_{z=\frac{K^2}{(2q\cdot K)}}$$

$$\boxed{A_n = \sum_{dia} \frac{A_L \otimes A_R}{K^2}\Bigg|_{z=\frac{K^2}{(2q\cdot K)}}}$$

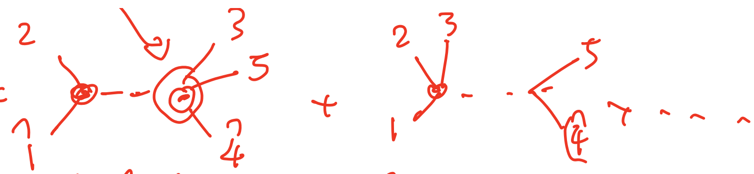
A_3

$$A_4(\hat{1}\,2\,3\,\hat{4}) = $$



$$\frac{A_3(\hat{1}\,2\,P)\,A_3(-P\,3\,\hat{4})}{s} + \frac{A_3(\hat{1}\,3\,P)\,A_3(-P\,2\,\hat{4})}{t}$$

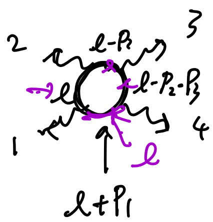
$A_5(\hat{1}2345\hat{5}) =$



$$\frac{A_3(\hat{1}2P)\,A_4(-P35\hat{4})}{(P_1+P_2)^2}$$

$A_6(\ldots)$

On-shell approach to loop corrections

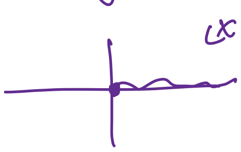


$\to \mathcal{A}_4^{L=1} =$ non-rational $\{\log(s),\ Li_2(\cdot,\cdot),\ \text{Elliptic functions}\}$

functions space

$$= \int \frac{d^D\ell}{(2\pi)^{D/2}}\, \frac{n(\ell, P_i, \lambda_i\tilde{\lambda}_i)}{(\ell+P_1)^2\,\ell^2\,(\ell-P_2)^2\,(\ell-P_2-P_3)^2}, \quad D=4$$

$$\log x = \int_1^x \frac{dy}{y}$$



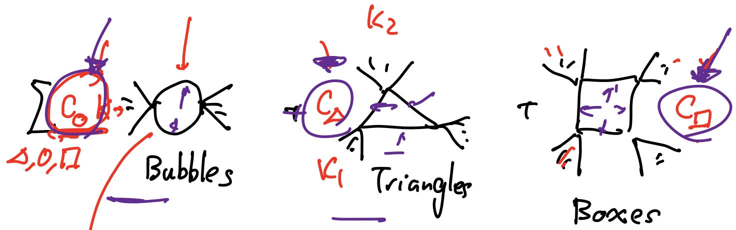
rational function

ambiguity 1: depends on what is ℓ

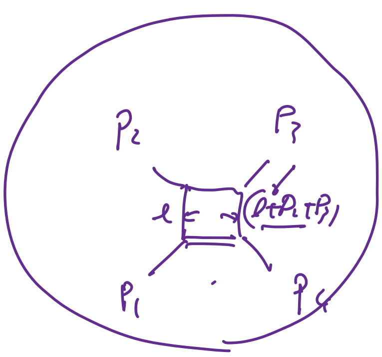
- - 2: IBP

Solution: Integral basis

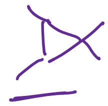
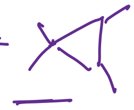
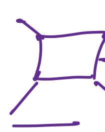
Claim: $\mathcal{A}_n^{L=1}\ (D=4) = \sum C_0$ Bubbles $+ C_\triangle$ Triangles $+ C_\square$ Boxes

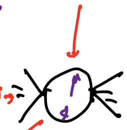
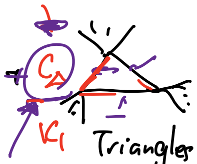


$$\int \frac{d^4\ell}{(2\pi)^{4/2}} \frac{1}{\ell^2(\ell-K)^2} \qquad \int d\ell \frac{1}{\ell^2(\ell-K_1)^2(\ell+K_2)^2} \qquad \int d\ell \frac{1}{(\;)(\;)(\;)(\;)}$$



$$\frac{\ell\cdot(P_2+P_3)}{(\;)(\;)(\;)(\;)} = \frac{(\ell+P_2+P_3)^2-\ell^2-(P_2+P_3)^2}{(\;)(\;)(\;)(\;)}$$

$=$  $+$  $+$  $(P_2+P_3)^2$

$\mathcal{A}_n^{L=1}\,(D=4) = \sum C_\circ$  $+ C_\triangle$  $+$  $C_\square$

$\Delta, \circ, \square$

Bubbles Triangles Boxes

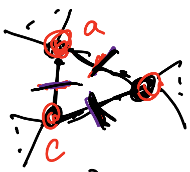
Unitary

$\boxed{C_\square}$ 

$C_\square = \sum A_a A_b A_c A_d$

Intermediate states

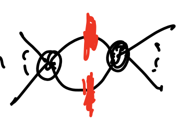
$\ell^2 = (\ell-P_1)^2 = (\quad) = (\quad) = m^2 \quad \ell = \ell^*$

$C_\triangle$ 

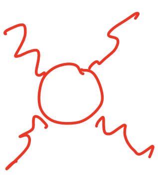
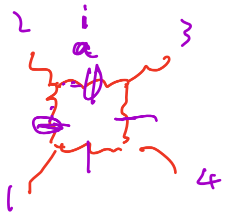
$\ell^2 = (\ell-P_1)^2 = (\ell-P_1-P_2)^2 = 0$

$\ell^*(z)$

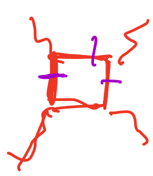
$$\sum_{\text{Intermediate state}} A_a A_b A_c\,(z)\Big|_{z\to\infty} = C_\triangle + \cancel{C_\square\, P^2}$$

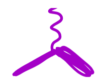
$C_\circ$ a  b

$$\sum_{\text{Intermediate}} A_a A_b\,(z,y)\Big|_{z,y\to\infty} = \boxed{C_\circ} + \cancel{C_\triangle \frac{1}{P^2}} + \cancel{C_\square \frac{1}{P_\alpha^2}\frac{1}{P_\beta^2}}$$

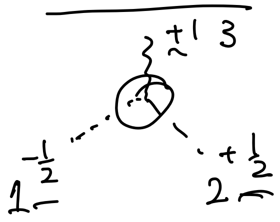
 $=$ $C_\square$ 

$C_\square = \sum_{\text{state}} A_3(1\ell)\, A_3(2\quad)\, A_3(3\quad)\, A_3(4\quad)$

$C_\square$  $= \sum_{\text{gluon}} A_3(1\quad) A_3(\quad) A_3(\quad) A_3 + \sum_{\text{fermion}}$

$A_3(\ )$. 

Exp: Chiral fermions coupled to YM SU(N)

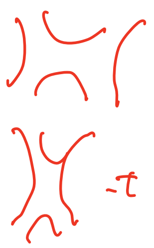


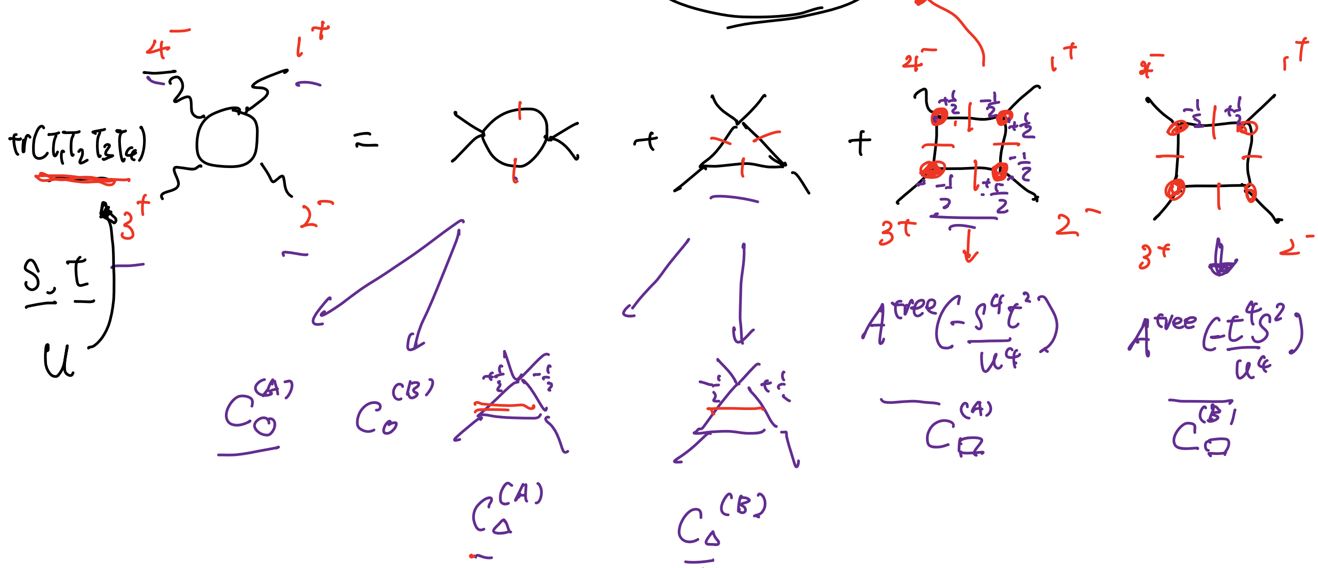
$$[12]^a\,[23]^b\,[31]^c$$

$$\frac{a}{2}+\frac{c}{2}=-\frac12$$
$$\frac{a}{2}+\frac{b}{2}=+\frac12 \quad\Big]\quad \frac{b-c}{2}=1$$
$$\frac{b}{2}+\frac{c}{2}=+1$$

$b=2$
$c=0$
$a=-1$

$$\frac{[23]^2}{[12]} \quad \checkmark$$

$\mathrm{tr}(TTTT)=$ 



$$\Rightarrow (A)+(B) = A^{tree}\times\left\{-\frac{st(t^2+t^2)}{2u^4}\left(\log\left(\frac{t}{s}\right)^2+\pi^2\right) + \left[\frac{s-t}{3u}-\frac{st(s-t)}{u^3}\right]\log\left(\frac{t}{s}\right) + \frac{1}{\epsilon^0}\right\}$$

Violates Locality

$= Answ_1$

$$(X)\ Answ_1 \big|_{u\to 0} = A_{tree} \times \left(-\frac{s^2}{u^2} - \frac{s}{u}\right) + \cdots$$

$$Answ_1 + R = Answ_2$$

$$A_{tree} \times \left(-\frac{st}{u^2}\right) \Big|_{u\to 0} = \frac{s^2}{u^2} + \frac{s}{u}$$

$$\boxed{Answ_1 + A_{tree}\left(-\frac{st}{u^2}\right)}$$

factors s t does not modify existing tree-level factorization

$$(A)-(B) = \overline{Answ_1}\big|_{u\to 0} = -\frac{s}{u} A_{tree}$$

$$\overline{Answ_1} + \overline{R} = \overline{Answ_2}$$

$$A_{tree}\,\frac{(s-t)}{2u}$$

$s=0$
$t=0$

enforcing Unitary + Locality

← quantum correction → tree-level exchange is modificat. → (longitudinal mode)

$$\Rightarrow d^{abc} f^{de}{}_{a} = 0$$

(anomaly cancellation condition